MATH2210Q - Quiz #1 Name: Key

For full credit, you must show all work and circle your final answer.

1a. Find the solution set to the following system of equations if it exists. (Write it in parametric form.)

$$\begin{aligned} x_1 + 3x_2 + 5x_3 &= 7 \\ 3x_1 + 5x_2 + 7x_3 &= 9 \\ 5x_1 + 7x_2 + 9x_3 &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 4 & 8 & 17 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

inconsistent system

1b. Are the following vectors linearly independent?

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} \right\}$$

From the above

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 0 \\ 3 & 5 & 7 & 0 \\ 5 & 7 & 9 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Hence, $A\vec{x} = \vec{0}$ where $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{bmatrix}$ has nontrivial solutions

and the vectors $\left\{ \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} \right\}$ are linearly dependent.

2 Let T be the following linear transformation.

$$T : \mathbb{R}^4 \to \mathbb{R}^3; \qquad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 4x_2 + 8x_3 + x_4 \\ x_2 - x_3 + 3x_4 \\ 5x_4 \end{bmatrix}$$

(a) Find the standard matrix for T.

$$A = \begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

(b) Determine if T is a one to one linear transformation.

The columns of A are $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}-4\\1\\0\end{bmatrix}, \begin{bmatrix}8\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\3\\5\end{bmatrix}\right\}$

which are linearly dependent since there are more vectors than entries. Hence, T is not 1 to 1.

3 (a) Compute $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} + 2 \begin{bmatrix} 3 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 7 & 6 & 3 \\ 5 & 2 & 5 \\ 5 & 2 & 2 \end{bmatrix}$$

(b) Compute $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 11 & 4 & 4 \\ 11 & 6 & 0 \\ 9 & 4 & 4 \end{bmatrix}$$