

For full credit, you must show all work and circle your final answer.

- 1 (a) Find the solution set to the following matrix equation. (Write it in parametric form.)

$$\begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 3 & -5 & 0 \\ -2 & 6 & 8 \\ 1 & 1 & 8 \end{array} \right] \sim \left[ \begin{array}{cc|c} 1 & 1 & 8 \\ 3 & -5 & 0 \\ -2 & 6 & 8 \end{array} \right] \sim \left[ \begin{array}{cc|c} 1 & 1 & 8 \\ 0 & -8 & -24 \\ 0 & 8 & 24 \end{array} \right] \sim \left[ \begin{array}{cc|c} 1 & 1 & 8 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right] \sim \left[ \begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \text{ is the only solution}$$

- (b) Find the solution set to the following vector equation. (Hint: Compare to the above.)

$$x_1 \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \end{bmatrix}$$

This vector equation is equivalent to the matrix equation above.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix} \text{ is the only solution.}$$

2 Determine which of the following sets of vectors are linearly independent.

(a)  $\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 9 \\ 0 \\ -1 \end{bmatrix} \right\}$  linearly independent  
 $\begin{bmatrix} 1 \\ 2 \\ 5 \\ 2 \end{bmatrix} \neq c \begin{bmatrix} 2 \\ 9 \\ 0 \\ -1 \end{bmatrix}$  for any  $c$  in  $\mathbb{R}$ .

(b)  $\left\{ \begin{bmatrix} 2 \\ -2 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ 2 \\ 5 \end{bmatrix} \right\}$

linearly dependent  
 contains the zero vector.

3 Determine if  $\mathbf{b}$  lies in the span of the given vectors.

$\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}; \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} \right\}$

$$\left[ \begin{array}{ccc|c} 2 & 0 & 6 & 4 \\ -1 & 8 & 5 & 1 \\ 1 & -2 & 1 & -4 \end{array} \right] \sim \left[ \begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 2 & 0 & 6 & 4 \\ -1 & 8 & 5 & 1 \end{array} \right] \sim \left[ \begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 0 & 4 & 4 & 12 \\ 0 & 6 & 6 & -3 \end{array} \right] \sim \left[ \begin{array}{ccc|c} 1 & -2 & 1 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 15 \end{array} \right] \sim \left[ \begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 15 \end{array} \right]$$

We have an inconsistent system

$\vec{\mathbf{b}}$  is not in the span.