

For full credit, you must show all work and circle your final answer.

1 Compute the determinant of the following matrices:

$$(a) A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

A is upper triangular
 $\det(A) = \prod_{i=1}^6 a_{ii} = 1$

$$(b) B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ 1 & 4 & 5 \end{bmatrix}$$

$$\begin{aligned} \det(B) &= -2 \det \begin{pmatrix} 3 & 2 \\ 1 & 5 \end{pmatrix} - 4 \det \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} \\ &= -2(13) - 4(-1) \\ &= -22 \end{aligned}$$

$$(c) C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$C \sim Id$ via two row swaps $R_2 \leftrightarrow R_3$ & $R_3 \leftrightarrow R_4$

$$\det(C) = (-1)^2 \det(Id) = 1$$

2 Find a **basis** for the following spaces corresponding to the matrix below.

$$D = \begin{bmatrix} 7 & -3 & 5 \\ -4 & 1 & -5 \\ -5 & 2 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -3 & 5 \\ -4 & 1 & -5 \\ -5 & 2 & -4 \end{bmatrix} \sim \begin{bmatrix} 7 & -3 & 5 \\ -4 & 1 & -5 \\ 5 & -2 & 4 \end{bmatrix} \sim \begin{bmatrix} 7 & -3 & 5 \\ -4 & 1 & -5 \\ 1 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -1 \\ -4 & 1 & -5 \\ 7 & -3 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -1 \\ 0 & -3 & -4 \\ 0 & 4 & 12 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) $\text{null}(D)$

$$[D|0] \sim \begin{bmatrix} 1 & 0 & 2 & | & 0 \\ 0 & 1 & 3 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \iff \begin{array}{l} x_1 + 2x_3 = 0 \\ x_2 + 3x_3 = 0 \\ x_3 = x_3 \end{array} \iff \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$$

$t \in \mathbb{R}$

$$\text{Basis} = \left\{ \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} \right\}$$

(b) $\text{col}(D)$

$$\text{Basis} = \left\{ \begin{bmatrix} 7 \\ -4 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} \right\}$$

(c) $\text{row}(D)$

$$\text{Basis} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \right\}$$