

Definitions to know:

- 1) least upper bound (pg. 23 book / pg. 12 notes)
- 2) greatest lower bound (pg. 25 book)
- 3) metric (pg. 34 book / pg. 16 notes)
- 4) metric space (pg. 34 book / pg. 16 notes)
- 5) open set (pg. 39 book / pg. 20 notes)
- 6) relatively open set (pg. 23 notes)
- 7) closed sets (pg. 40 book / pg. 23 notes)
- 8) interior of a set (pg. 25 notes)
- 9) closure of a set (pg. 25 notes)
- 10) boundary of a set (pg. 26 notes)
- 11) a bounded set (pg. 43 book)
- 12) sequence (pg. 11 book / pg. 30 notes)
- 13) a sequence converging (pg. 45 book / pg. 30 notes)
- 14) limit point (pg. 55 book / pg. 32 notes)
- 15) an increasing sequence (pg. 50 book / pg. 33 notes)
- 16) a decreasing sequence (pg. 50 book / pg. 33 notes)
- 17) a strictly increasing sequence (pg. 33 notes)
- 18) a strictly decreasing sequence (pg. 33 notes)
- 19) a bounded sequence (pg. 47 book / pg. 33 notes)
- 20) a subsequence (pg. 46 book / pg. 37 notes)
- 21) \liminf (pg. 38 notes)
- 22) \limsup (pg. 38 notes)
- 23) Cauchy sequence (pg. 51 book / pg. 44 notes)
- 24) complete space (pg. 52 book / pg. 45 notes)
- 25) compact set (pg. 54 book / pg. 49 notes)

Proofs to know:

1) Pg. 39 book. Pg. 21 notes

Theorem 1. *For any metric space X the following are true:*

- i) \emptyset and X are open.*
- ii) the union of any collection of open subsets of X is open.*
- iii) the intersection of a finite number of open subsets of X is open.*

2) Pg. 47 book. Pg. 32 notes.

Theorem 2. *Suppose X is a metric space, $S \subset X$. A set is closed if and only if every sequence from S which converges in X actually converges in S*

3) Pg. 50 book. Pg. 32 notes.

Theorem 3. *If (a_n) is an increasing sequence from \mathbb{R} , then $(a_n)_{n=n_0}^{\infty}$ converges if and only if it is bounded above. If it is bounded above then it converges to*

$$\sup\{a_n : n \geq n_0\}$$

If it is not bounded above, then it diverges to infinity.

4) Pg. 52 book.

Theorem 4. *A Cauchy sequence that has a convergent subsequence is itself convergent*

5) Pg. 51 book.

Theorem 5. *A convergent sequence is a Cauchy sequence.*

6) Pg. 52 notes.

Theorem 6. *A closed bounded subset of \mathbb{R} is compact.*