1. Let $p_n = a_m n^m + a_{m-1} n^{m-1} + \ldots + a_1 n + a_0$ and $q_n = b_m n^m + b_{m-1} n^{m-1} + \ldots + b_1 n + b_0$ for $n > 0$. Prove that
$$\lim_{n \to \infty} \frac{p_n}{q_n} = \frac{a_m}{b_m}.$$

2. Suppose that y is a limit point of a metric space X. Show that $Y = X \setminus \{y\}$ is not complete.

3. Let (X, d_X) and (Y, d_Y) be complete metric spaces. Let $(X \times Y, d)$ be the metric space defined by the metric
$$d : (X \times Y) \times (X \times Y) \to \mathbb{R}; \qquad d((x, y), (a, b)) = d_X(x, a) + d_Y(y, b)$$
is a complete metric space.

4. Let S be a bounded subset of $\mathbb{R}$. Show that
$$\inf(S) = -\sup(-S)$$
where $-S = \{-s \ : \ s \in S\}$.

5. A metric space (X, d) is called sequentially compact if every sequence has a convergent subsequence. Show that X is sequentially compact if and only if every infinite subset has a limit point in X.