MATH 3210 Your name
Homework 1 January 19, 2018

1. Let $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$. Note that $\mathbb{Q}(\sqrt{2})$ is field and more specifically it is known as an algebraic number field. The binary operations on $\mathbb{Q}(\sqrt{2})$ are the standard addition and multiplication of numbers. Verify for all $\alpha \neq 0$ in $\mathbb{Q}(\sqrt{2})$ that there exists a $\beta \in \mathbb{Q}(\sqrt{2})$ such that $\alpha \cdot \beta = 1$.

2. Is the space of non-negative functions on the interval $[0, 1]$ a vector space over the real numbers $\mathbb{R}$? Justify your answer with a proof.

3. Let $M_{2\times 2}$ be the set of $2 \times 2$ matrices with real entries, i.e.

$$M_{2\times 2} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a, b, c, d \in \mathbb{R} \right\}.$$

$M_{2\times 2}$ is a vector space over the reals with the operations

$$k \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix} \text{ with } k \in \mathbb{R}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} = \begin{pmatrix} a + a' & b + b' \\ c + c' & d + d' \end{pmatrix}$$

Identify the additive identity in $M_{2\times 2}$ and justify your answer with a proof.

4. Are the positive real numbers a field? Justify your answer.

5. Suppose $a \in \mathbb{F}$, $v \in V$, and $av = 0$. Prove $a = 0$ or $v = 0$.