MATH 3210 Your name
Homework 3 date

For this homework assume all matrices are square and have entries from a field $\mathbb{F}$.

1. Show by induction that the determinant of an upper triangular matrix is the product of the diagonal entries.

2. Call a matrix A nilpotent if $A^k = 0$ for some positive integer k. Show that every square nilpotent matrix has determinant zero.

3. Let A be an $n \times n$ matrix. Show from the definition of determinants that

$$\det(kA) = k^n \det(A)$$

for $k \in \mathbb{F}$.

4. Let $T : V \to V$ where V is a finite dimensional vector space. Let $\beta = \{v_1, \ldots, v_n\}$ and $\gamma = \{v'_1, \ldots, v'_n\}$ be two ordered bases for V. Show there exists an invertible matrix P such that

$$[T]_\beta = P^{-1}[T]_\gamma P$$

Hint: Construct a linear transformation that switches out the basis and figure out how it relates to the matrix construction.

5. Define the determinant of a linear transformation $T : V \to V$ by

$$\det(T) = \det([T]_\beta)$$

where $[T]_\beta$ is a matrix of T with respect to an ordered basis. Does this definition rely on the choice of basis ?