MTH130 - Practice Sheet **Name:**

Find the derivatives of the functions below.

1) $f(x) = 5$

$f'(x) = 0$

2) $f(x) = x^{3.2}$

$f'(x) = 3.2x^{2.2}$

3) $f(x) = x^2 + 3x + 1$

$f'(x) = 2x + 3$

4) $f(x) = 3x^7 + 2x^6 + 8x^5 + 2x^4 + x^3 - 5x^2 - x - 81$

$f'(x) = 21x^6 + 12x^5 + 40x^4 + 8x^3 + 3x^2 - 10x - 1$

5) Find the tangent line to the curve $f(x) = x^2 + 4$ at $x = 2$

$(2, f(2)) = (2, 8)$

$y - 8 = 4(x-2)$

$f'(x) = 2x \quad f'(2) = 4$

Use the product rule to calculate the derivatives below.

1) $f(x) = x^2$, $g(x) = 3x^4$. Find $\frac{d}{dx}(f(x) \cdot g(x))$

$x^2(12x^3) + (3x^4)(2x) = 12x^5 + 6x^5 = 18x^5$

2) $f(x) = x^2 - x + 1$, $g(x) = x^3 + \sqrt{x}$. Find $\frac{d}{dx}(f(x) \cdot g(x))$

$(x^2 - x + 1)(3x^2 + \frac{1}{2}x^{-1/2}) + (x^3 + \sqrt{x})(2x - 1)$

3) $f(x) = x^{3/4}$, $g(x) = x^{1/4}$. Find $\frac{d}{dx}(f(x) \cdot g(x))$

$x^{3/4} \, \frac{1}{4}x^{-3/4} + x^{1/4} \cdot \frac{3}{4}x^{-1/4} = 1$

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Use the quotient rule to calculate the derivatives below.

1) $f(x) = x^2$, $g(x) = 3x^4$. Find $\frac{d}{dx}(f(x)/g(x))$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{3x^4(2x) - x^2(12x^3)}{(3x^4)^2} = \frac{6x^5 - 12x^5}{9x^8} = \frac{-6}{9}x^{-3}$$

$$= \frac{-1}{3x^3}$$

2) $f(x) = x^2 - x + 1$, $g(x) = x^3 + \sqrt{x}$. Find $\frac{d}{dx}(f(x)/g(x))$

$$\frac{(x^3 + x^{1/2})(2x-1) - (x^2 - x + 1)(3x^2 + \frac{1}{2}x^{-1/2})}{(x^3 + \sqrt{x})^2}$$

$$= \frac{(2x^4 + 2x^{3/2} - x^3 - x^{1/2}) - (3x^4 - 3x^3 + 3x^2 + \frac{1}{2}x^{3/2} - \frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2})}{(x^9 + 2x^{7/2} + x)}$$

3) $f(x) = x^{3/4}$, $g(x) = x^{1/4}$. Find $\frac{d}{dx}(f(x)/g(x))$

$$\frac{x^{1/4} \cdot (\frac{3}{4}x^{-1/4}) - x^{3/4}(\frac{1}{4}x^{-3/4})}{x^{2/4}} = \frac{\frac{3}{4} - \frac{1}{4}}{x^{1/2}} = \frac{1}{2\sqrt{x}}$$

MTH151 - Practice Sheet Name: Soln

1) $\int e^{-5r}\,dr$

Let $u = -5r$

$du = -5\,dr$

$\Rightarrow -\frac{1}{5}\,du = dr$

$$\int e^{-5r}\,dr = -\frac{1}{5}\int e^{u}\,du = -\frac{1}{5}e^{u} + C = -\frac{1}{5}e^{-5r} + C$$

2) $\int e^x\sqrt{1+e^x}\,dx$

Let $u = 1+e^x$

$du = e^x\,dx$

$$\int e^x\sqrt{1+e^x}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{3}(1+e^x)^{3/2} + C$$

3) $\int \cot(x)\,dx$ Note $\cot(\theta) = \frac{\cos\theta}{\sin\theta}$

Let $u = \sin x$

$du = \cos x\,dx$

$$\int \cot(x)\,dx = \int \frac{\cos x}{\sin x}\,dx = \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sin x| + C$$

4) $\int \frac{x}{x^2+4}\,dx$

Let $u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$$\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+4| + C$$

5) $\int \frac{x}{1+x^4}\,dx$

Let $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$$\int \frac{x\,dx}{1+x^4} = \frac{1}{2}\int \frac{1}{1+u^2}\,du$$

$$= \frac{1}{2}\arctan(u) + C$$

$$= \frac{1}{2}\arctan(x^2) + C$$

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6) $\int (x-1)\sin(x)\,dx$

$u = x-1 \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$$\int (x-1)\sin x\,dx = -(x-1)\cos x + \int \cos x\,dx$$

$$= -(x-1)\cos x + \sin x + C$$

7) $\int x\tan^2(x)\,dx$

$u = x \quad dv = \tan^2(x) = \sec^2(x) - 1$

$du = dx \quad v = \tan x - x$

$$\int x\tan^2(x)\,dx = x\tan x - x^2 - \int \tan x - x \; dx$$

$$= x\tan x - x^2 - \left[\ln|\sec x| - \frac{x^2}{2}\right] + C$$

$$= x\tan x - \frac{x^2}{2} - \ln|\sec x| + C$$

8) $\int (\arcsin(x))^2\,dx$

$u = (\arcsin(x))^2 \quad dv = dx$

$du = \dfrac{2(\arcsin(x))}{\sqrt{1-x^2}} \quad v = x$

$$\int (\arcsin(x))^2\,dx = x(\arcsin(x))^2 - \int \frac{2x(\arcsin(x))}{\sqrt{1-x^2}}\,dx$$

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$u = \arcsin(x) \quad dv = \dfrac{2x}{\sqrt{1-x^2}}$

$du = \dfrac{1}{\sqrt{1-x^2}}\,dx \quad v = -2(1-x^2)^{\frac{1}{2}}$

$$= x(\arcsin(x))^2 - \left[-2\sqrt{1-x^2}\arcsin x + \int \frac{2\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx\right]$$

$$\boxed{= x(\arcsin(x))^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C}$$