MTH151 - Practice Sheet Name: Soln

1) $\int e^{-5r}\, dr$

Let $u = -5r$

$du = -5\, dr$

$\Rightarrow \frac{-1}{5} du = dr$

$$\int e^{-5r} dr = \frac{-1}{5}\int e^{u} du = \frac{-1}{5} e^{u} + C = \frac{-1}{5} e^{-5r} + C$$

2) $\int e^x\sqrt{1+e^x}\, dx$

Let $u = 1+e^x$

$du = e^x dx$

$$\int e^x\sqrt{1+e^x}\, dx = \int \sqrt{u}\, du = \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{3}(1+e^x)^{3/2} + C$$

3) $\int \cot(x)\, dx$ Note $\cot(\theta) = \frac{\cos\theta}{\sin\theta}$

Let $u = \sin x$

$du = \cos x\, dx$

$$\int \cot(x)\, dx = \int \frac{\cos x}{\sin x} dx = \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|\sin x| + C$$

4) $\int \frac{x}{x^2+4}\, dx$

Let $u = x^2+4$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$$\int \frac{x}{x^2+4} dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+4| + C$$

5) $\int \frac{x}{1+x^4}\, dx$

Let $u = x^2$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$$\int \frac{x\, dx}{1+x^4} = \frac{1}{2}\int \frac{1}{1+u^2} du$$

$$= \frac{1}{2}\arctan(u) + C$$

$$= \frac{1}{2}\arctan(x^2) + C$$

MTH151 - Practice Sheet **Name:**

6) $\int (x-1)\sin(x)\,dx$

$u = x-1 \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$\int (x-1)\sin x\,dx = -(x-1)\cos x + \int \cos x\,dx$

$= -(x-1)\cos x + \sin x + C$

7) $\int x\tan^2(x)\,dx$

$u = x \quad dv = \tan^2(x) = \sec^2(x) - 1$

$du = dx \quad v = \tan x - x$

$\int x\tan^2(x)\,dx = x\tan x - x^2 - \int \tan x - x \; dx$

$= x\tan x - x^2 - \left[\ln|\sec x| - \frac{x^2}{2}\right] + C$

$= x\tan x - \frac{x^2}{2} - \ln|\sec x| + C$

8) $\int (\arcsin(x))^2\,dx$

$u = (\arcsin(x))^2 \quad dv = dx$

$du = \underbrace{\frac{2(\arcsin(x))}{\sqrt{1-x^2}}} \quad v = x$

a 2nd IBP

$u = \arcsin(x) \quad dv = \frac{2x}{\sqrt{1-x^2}}$

$du = \frac{1}{\sqrt{1-x^2}}dx \quad v = -2(1-x^2)^{\frac{1}{2}}$

$\int (\arcsin(x))^2\,dx = x(\arcsin(x))^2 - \int \frac{2x(\arcsin(x))}{\sqrt{1-x^2}}\,dx$

$= x(\arcsin(x))^2 - \left[-2\sqrt{1-x^2}\arcsin x + \int \frac{2\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx\right]$

$= x(\arcsin(x))^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C$