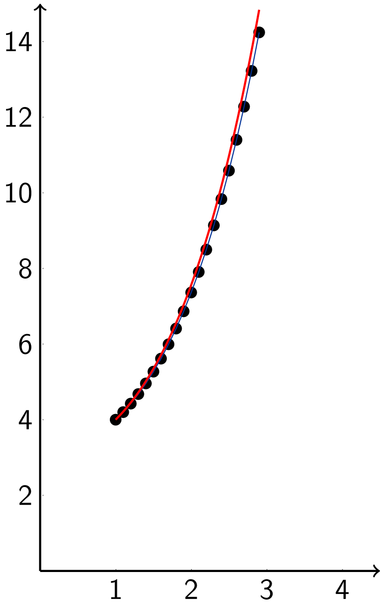


Here we apply Euler's Method to the following IVP:

$$\frac{dy}{dx} = x\sqrt{y}, \qquad y(1) = 4$$

Recursion Formulas:

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Actual Solution:

$$y = \frac{(x^2+7)^2}{16}$$

$$x_{n+1} = x_n + h; \qquad y_{n+1} = y_n + hf(x_n, y_n); \qquad h = 0.1$$

$$x_0 = 1; \qquad y_0 = 4$$

$$x_1 = x_0 + .1 = 1.1; \qquad y_1 = y_0 + (0.1)(1)\sqrt{4} = 4.2$$

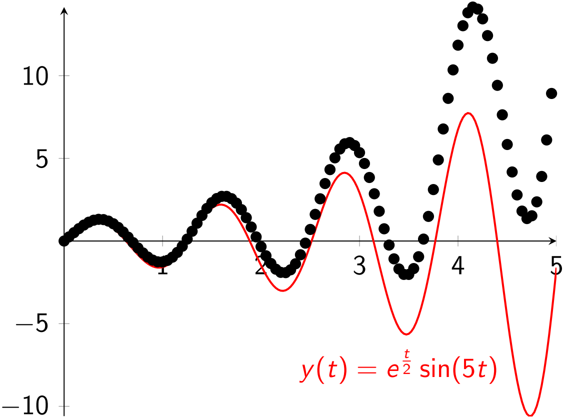
$$x_2 = x_1 + .1 = 1.2; \qquad y_2 = y_1 + (0.1)(1.1)\sqrt{4.2} \approx 4.425$$

$$x_3 = x_2 + .1 = 1.3; \qquad y_3 = y_2 + (0.1)(1.2)\sqrt{4.425} \approx 4.678$$

$$x_4 = x_3 + .1 = 1.4; \qquad y_4 = y_3 + (0.1)(1.3)\sqrt{4.678} \approx 4.959$$

Eulers Method	Exact Value	Percent Error
4	4	0
4.20000000000000	4.21275625000000	0.302800571478606
4.42543291685111	4.45210000000000	0.598977631879118
4.67787347223526	4.71975625000000	0.887392813235733
4.95904257025418	5.01760000000000	1.16704061196227
5.27080728021505	5.34765625000000	1.43705889444475
5.61518086196144	5.71210000000000	1.69673391639791
5.99432282754453	6.11325625000000	1.94550036170118

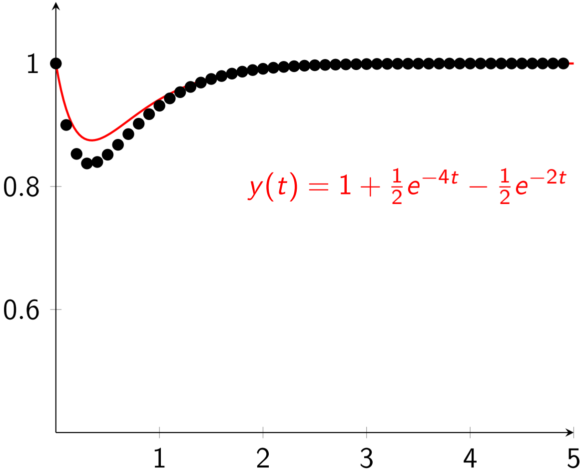
As we go father away from the start point the approximation gets worse.



Here we used Euler's Method with a step size of h=0.05 on the following differential equation:

$$y' - y = -\frac{1}{2}e^{\frac{t}{2}} \sin(5t) + 5e^{\frac{t}{2}} \cos(5t); \quad y(0) = 0$$

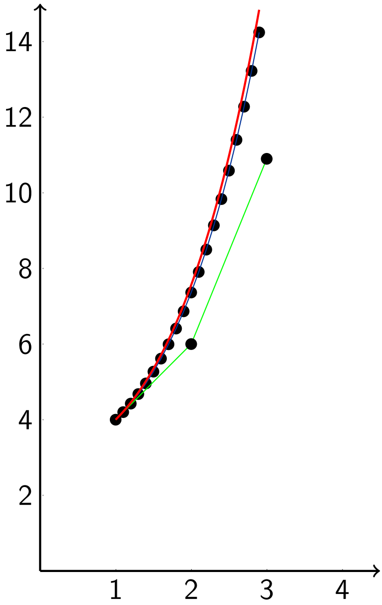
Again as we go father away the estimation of the solution becomes worse.



Here we applied Euler's Method to

$$y' + 2y = 2 - e^{-4t} \qquad y(0) = 1$$

Notice the approximation is worse where the solution changes rapidly.



We return to our first example:

$$\frac{dy}{dx} = x\sqrt{y}, \qquad y(1) = 4$$

To illustrate the importance of step-size we have graphed the solution along with an Euler's method approximation with a step-size of $h = .1$ and an approximation with a step-size of $h = 1$.

In general the smaller the step-size the better the approximation.